

STATIONARY STATES OF A BINARY FLUIDIZED BED

Yu. A. Buevich and Sh. K. Kapbasov

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Steady-state vertical flows and fluidized beds of a polydisperse material are considered based on the hypothesis on the collisional mechanism of energy and momentum exchange between particles. Representations are obtained for the effective energy of particle pulsations and the partial pressures, produced by the latter, in a binary fluidized bed. A stationary particle distribution with respect to the height of the bed is investigated.

The majority of available physicomathematical models and design methods pertain to monodisperse flows and fluidized beds [1]. In some cases such models and methods are also successfully used to describe real polydisperse mixtures of particles. However there are situations in which polydispersity is of importance or even of primary importance, whereas the indicated methods turn out to be insufficient. This especially refers to the process of gravitational separation of ores and other lump- or powder-like materials [2], processing of disperse systems in fluidized beds with inert particles (drying [3], annealing [4], and so on), to low-temperature combustion of sprayed solid or liquid fuel [5, 6], to pneumo- and hydrotransport of polydisperse materials [7], etc.

The behavior of polydisperse flows and fluidized beds differs in many respects from that of analogous monodisperse systems. This is particularly pronounced in the case of segregation of different fractions of a disperse material along the height of the fluidized bed, whose nonstandard character has been noted in [8, 9]. Those effects as well as mutual motion of the fractions in polydisperse flows exert a crucial influence on the effectiveness of separation processes as well as on the structure of beds and flows itself and therefore they have been intensely studied experimentally under laboratory and industrial conditions (see the examples in [10-19]). However, the theoretical models of the effects and processes discussed are based on a number of purely empirical assumptions and do not favor either clearer understanding of the phenomena observed in transport and fluidization of polydisperse materials or development of reliable design methods.

In the present work polydisperse systems are modeled by generalizing the theory of monodisperse gas suspensions recently set forth in [20, 21]. The general results are illustrated by the example of a fluidized bed having coarse particles only of two species that differ in their size and density.

Background of the Model. We will consider a mixture of spherical particles distributed with respect to their radius a . The material density of the particles d_1 is a function of their size in the general case. We assume the density $n(a)$ of the size distribution of the number of particles to be normalized to the total numerical concentration n .

The particles are drawn into random pseudoturbulent motion, whose characteristics depend on the mechanism of energy and momentum exchange between the particles. If the latter are not too small, this exchange is primarily accomplished via direct particle-particle collisions. In this case, the pseudoturbulence in a monodisperse system must be isotropic according to the principle of equidistribution of energy with respect to translational degrees of freedom of particle pulsations. Here the rotational energy of the particles may be neglected [20, 21].

In a polydisperse system, from this principle the equality of the average kinetic energies per degree of freedom for particles of different size and density is inferred. Therefore if we introduce, in the usual way, a "temperature" θ of pulsations as the doubled average energy of one degree of freedom, this temperature must be the same for particles of different types, i.e.,

$$m(a) \langle \omega'^2(a) \rangle = \langle m \rangle \langle \omega'^2 \rangle = 3\theta, \quad (1)$$

where w' is some random vector-function, while average mass of the particles is

$$\langle m \rangle = \frac{1}{n} \int m(a) n(a) da, \quad m(a) = \sigma(a) d_1(a), \quad \sigma(a) = \frac{4\pi}{3} a^3. \quad (2)$$

Next, we pass from analyzing the random velocities $w'(a)$ of particles of different types to investigation of the random unit vector w' . Based on Eq. (1), it is natural to assume that

$$w'(a) = (\langle m \rangle / m(a))^{1/2} w'. \quad (3)$$

Considering the particles to be sufficiently coarse, we use the result from [22] to describe the force of their hydrodynamic interaction with the carrier-gas flow. Then the total force acting on a single particle of given radius with an account of its weight and the buoyancy effect may be written in the following form:

$$\begin{aligned} f(a) &= m(a) \left\{ \frac{1}{a\kappa(a)} K(\rho) u(a) u(a) + \left[1 - \frac{\varepsilon}{\kappa(a)} - \rho \frac{\langle d_1 \rangle}{d_1(a)} \right] g \right\}, \\ K(\rho) &= \frac{3\xi}{8} \left(\frac{1-\rho}{1-1.17\rho^{2/3}} \right)^2, \quad \kappa(a) = \frac{d_1(a)}{d_0}, \quad \varepsilon = 1 - \rho, \end{aligned} \quad (4)$$

where the average density of the solid phase is introduced analogously to Eq. (2):

$$\langle d_1 \rangle = \frac{\langle m \rangle}{\langle \sigma \rangle}, \quad \langle \sigma \rangle = \frac{1}{n} \int \sigma(a) n(a) da = \frac{1}{n} \int \rho(a) da. \quad (5)$$

If the particles differ not only in their size but in density as well, quantity (5) may fluctuate. However for the sake of simplicity, we will consider it to be a deterministic local characteristic of the disperse medium, not subjected to fluctuations, because the variations in $\langle d_1 \rangle$ can be related only to large-scale fluctuations, whereas the fluctuations responsible for pseudoturbulence initiation pertain to individual particles [20, 21].

We note that the gas slip velocity $u(a)$ relative to different particles is not the same. This statement, trivial for pulsations, is also valid in some cases for average motion (e.g., under pneumotransport conditions or in the stationary fluidized beds with continuous charging and discharging of a dispersed material).

The effect of pulsations on the average force of interphase interaction in the system turns out to be rather weak [21]. Therefore to a first approximation it may be neglected. Then Eq. (4) will also describe the indicated force if by $u(a)$ and ρ (as well as ε) are meant the corresponding average quantities. The pulsation of the force with an accuracy to quantities of first order in the fluctuations is

$$\begin{aligned} f'(a) &= m(a) \left\{ \frac{1}{a\kappa(a)} \left[K(\rho) u(a) (v' - w'(a)) + \right. \right. \\ &+ K(\rho) [u_0(v' - w'(a))] u(a) + \frac{dK}{d\rho} u(a) u(a) \rho' \left. \right] - \\ &\left. - \left[\frac{\langle d_1 \rangle}{d_1(a)} - \frac{1}{\kappa(a)} \right] g \rho' \right\}, \quad u_0 = \frac{u(a)}{u(a)}. \end{aligned} \quad (6)$$

Here variables without primes are understood as average quantities. For the sake of simplicity, it is assumed that the direction $u(a)$ does not depend on a . In gas-fluidized systems, $\kappa(a) \gg 1$, so that the second term in brackets at g may be ignored.

In what follows, in order to determine θ and other quantities, we must consider stochastic equations for random pulsations, as has been done in [21] for monodisperse systems. Closing the solution of such equations requires the variance $\langle \rho'^2 \rangle$ of the volume concentration fluctuations of a polydisperse system. The latter may be found, as in [20], using the statistical theory of thermodynamic fluctuations [23]. As a result, if the initial polydisperse mixture of particles is broken down into a finite number of fractions with the characteristic dimensions

a_1, a_2, \dots , then expressions may be obtained for a $\langle \rho'(a_i)\rho'(a_j) \rangle$ -type correlation in terms of second-order derivatives of any thermodynamic potential of the particle system using whole numbers of particles of different types. Then it is easy to determine $\langle \rho'^2 \rangle$ by conventional methods. If necessary, we may pass, in the limit, to systems with continuous size distribution by increasing without bound the number of fractions.

However, even for a binary disperse system (having only two fractions) concrete calculations are too cumbersome. At the same time in the general case $\langle \rho'^2 \rangle$ does not differ much from its counterpart for a monodisperse system. Therefore it would be reasonable to use the latter quantity as an appropriate approximation. If we employ, as in [21], the Carnahan-Starling model [24] of a system of identical spheres, then we have [25]

$$\langle \rho'^2 \rangle = \rho^2 \left[1 + 2\rho \frac{4 - \rho}{(1 - \rho)^4} \right]^{-1}. \quad (7)$$

Within the framework of the same approximation

$$\langle \rho'^2(a) \rangle = \frac{\rho(a)}{\rho} \langle \rho'^2 \rangle = \rho \left[1 + 2\rho \frac{4 - \rho}{(1 - \rho)^4} \right]^{-1} \rho(a). \quad (8)$$

In states near the close-packing one, the formula for $\langle \rho'^2 \rangle$, following from the Enskog theory of dense gases of identical spherical particles [25], may give better results. However in those states the error due to use of a fictitious monodisperse system instead of a real polydisperse one must increase. For definiteness we use Eqs. (7) and (8) below within the entire range of the total volume concentration ρ .

Stochastic Equations. The Langevin equation for a particle of given radius in the coordinate system, associated with its average motion, and with the assumption of relative smallness of its velocity fluctuations may be written in the form [21]

$$m(a) (\partial \mathbf{w}'(a) / \partial t) = \mathbf{f}'(a) + \mathbf{f}_c(a), \quad (9)$$

where $\mathbf{f}_c(a)$ is the force acting on a chosen particle in collisions with other particles. It may be represented as a sum of short pulses of random amplitude randomly distributed along the time axis. In [21], using, instead of Eq. (9), an equation similar in form was proposed, averaged over the ensemble of physically permissible realizations of the random environment of the chosen particle, having a random velocity $\mathbf{w}'(a)$. But even under this condition the indicated force must be a result of collisions with particles of all other sizes in a polydisperse system, i.e., must be expressed in terms of some functional of $\mathbf{w}'(a)$. As a consequence, the equation obtained from (9) will be integrodifferential.

To overcome this difficulty, we sum Eq. (9) over all particles with weight $n(a)$, assuming at the same time that averaging over the indicated ensemble has been done, i.e., that Eq. (9) contains the already smoothed collisional force. For simplicity, we shall consider here only situations in which the average gas flow velocity is the same for all particles, which allows us, in writing Eq. (9), to use the same coordinate system for particles of different size. Then from Eq. (9) with account for Eqs. (3), (5), and (6) we obtain

$$\gamma_1 (\partial \mathbf{w}' / \partial t) = \mathbf{f}' / \langle m \rangle + \mathbf{f}_c / \langle m \rangle, \quad (10)$$

where

$$\begin{aligned} \mathbf{f}' / \langle m \rangle = & F [\gamma_2 \mathbf{v}' - \gamma_3 \mathbf{w}' + \gamma_2 (\mathbf{u}_0 \mathbf{v}') \mathbf{u} - \gamma_3 (\mathbf{u}_0 \mathbf{w}') \mathbf{u}] + \\ & + [\gamma_2 (dF/d\rho) \mathbf{u} \mathbf{u} - \mathbf{g}] \rho', \quad F = K/a^3 \kappa^3, \end{aligned} \quad (11)$$

whereas for the collisional force per unit mass, with use of the same considerations as in [21], it may be assumed that

$$\mathbf{f}_c / \langle m \rangle = -A \mathbf{w}' - B (\mathbf{u}_0 \mathbf{w}') \mathbf{u}_0. \quad (12)$$

Here the coefficients

$$\begin{aligned}
\gamma_1 &= \frac{1}{n \langle m \rangle^{1/2}} \int m^{1/2}(a) n(a) da, \\
\gamma_2 &= \frac{a^\circ \kappa^\circ}{n \langle m \rangle} \int \frac{m(a)}{a \kappa(a)} n(a) da, \\
\gamma_3 &= \frac{a^\circ \kappa^\circ}{n \langle m \rangle^{1/2}} \int \frac{m^{1/2}(a)}{a \kappa(a)} n(a) da,
\end{aligned} \tag{13}$$

and the characteristic values a° and κ° are introduced. In a monodisperse system, it is natural to assume that the latter are equal to the real particle radius and the phase density ratio, so that all γ_j become unity in this case. The coefficients A and B in Eq. (12) should be defined from the isotropy conditions of pseudoturbulent pulsations of the particles and the equality of the work of all collisional forces in the system of the total dissipation of energy in collisions, as has been done in [21].

Equations (10) must be supplemented with the equations

$$(\partial/\partial t + \mathbf{u}\nabla)\rho' = \varepsilon\nabla v', \quad 0 = -\nabla\rho' - n\mathbf{f}', \tag{14}$$

following from the laws of mass and momentum conservation for the gas. Here the force of interphase interaction per unit volume of the mixture appears, which is determined, in fact, in Eq. (11); use is made of the previous coordinate system associated with the average motion of the disperse phase.

As in [21], we employ the correlation theory of stationary random processes, according to which the pulsations are represented in the form of stochastic Fourier-Stieltjes integrals over frequencies and wave space, containing the corresponding random measures. Then system of equations (10), (14) is transformed to a system of linear algebraic equations for the random measures. It is easy though rather tedious to solve the latter. In particular, for the random measures of the longitudinal (with respect to \mathbf{u}) and one of the transverse components of the pulsation velocity \mathbf{w}' we have

$$\begin{aligned}
dZ_{w_1} &= \frac{k_1^2}{k^2} \frac{xMu}{[x + (1-t^2)y]\gamma + (2-t^2)x(x+y)} dZ_\rho, \\
dZ_{w_2} &= \frac{k_1 k_2}{k^2} \frac{(x+y)Mu}{[x + (1-t^2)]\gamma + (2-t^2)x(x+y)} dZ_\rho, \quad t^2 = k_1^2/k^2,
\end{aligned} \tag{15}$$

where the first coordinate axis is chosen along \mathbf{u} , while dZ_ρ is the random measure of the concentration fluctuations. Furthermore, the following notation is introduced:

$$\begin{aligned}
x &= \frac{A}{2\gamma_2 Fu}, \quad y = \frac{B}{2\gamma_2 Fu}, \quad \gamma = \frac{\gamma_3}{\gamma_2}, \\
M &= \frac{1}{\varepsilon} + \frac{1}{2\gamma_2 Fu^2} \left(\gamma_2 \frac{dF}{d\rho} u^2 + g \right), \quad g = -g\mathbf{u}_0.
\end{aligned} \tag{16}$$

The random measures may be used to express the spectral densities of different quantities in terms of the spectral density of the concentration fluctuations which after integration over frequencies and wave space give the corresponding average values. Using the same representation for the spectral density of the total-concentration fluctuations as in [21], we have from Eq. (15)

$$\begin{aligned}
\langle \omega_1'^2 \rangle &= \left[\frac{x}{\gamma y + x(x+y)} \right]^2 (Mu)^2 \langle \rho'^2 \rangle \int_0^1 \frac{t^4 dt}{(b-t^2)^2}, \\
\langle \omega_2'^2 \rangle &= \left[\frac{x+y}{\gamma y + x(x+y)} \right]^2 (Mu)^2 \langle \rho'^2 \rangle \frac{1}{2} \int_0^1 \frac{t^2(1-t^2) dt}{(b-t^2)^2},
\end{aligned}$$

$$b = \frac{(2x + \gamma)(x + y)}{\gamma y + x(x + y)}, \quad (17)$$

where $\langle \rho'^2 \rangle$ is defined in Eq. (7), while γ and M are given in Eq. (16). For a monodisperse system ($\gamma_2 = \gamma_3 = 1$), these relations naturally convert those obtained in [21].

Now we neglect the energy dissipation of the system due to collisions between the particles. Then the average work of the collisional forces must vanish and, as a consequence, we arrive at the previous result: $y = -3x$ [21]. The isotropy condition is easily written on the basis of formulas (17), thus allowing determination of x . Finally we have

$$\sqrt{b} = 1,598, \quad x = -5,122\gamma, \quad y = 15,366\gamma. \quad (18)$$

and furthermore

$$\langle \omega_1'^2 \rangle = \langle \omega_2'^2 \rangle = C(M/\gamma)^2 u^2 \langle \rho'^2 \rangle, \quad C = 1,17 \cdot 10^{-3}. \quad (19)$$

It is pertinent to note that these results are obtained using the same assumption on the relative smallness of the frequency of pulsations as for the analogous results in [21].

Temperature of Pulsations and Partial Pressures of the Particles. We now write the basic relations characterizing the effect of random pulsations of particles on the averaged state of the mixture under consideration. The temperature of pulsations introduced in Eq. (1) is expressed on the basis of Eq. (19) as

$$\theta = \langle m \rangle \langle \omega_1'^2 \rangle = 1,17 \cdot 10^{-3} (M/\gamma)^2 \langle \rho'^2 \rangle \langle m \rangle u^2. \quad (20)$$

The partial pressure of the particles with sizes ranging from a to $a + da$ is represented in the form

$$p_1(a) da = G(\rho) \theta n(a) da, \quad G(\rho) = \frac{1 + \rho + \rho^2 - \rho^3}{(1 - \rho)^3}, \quad (21)$$

where $G(\rho)$ is the correction function appearing in the equation of state of solid spheres in conformity with the model of [24].

The equation describing the macroscopic distribution of the particles in the situation considered is written, with account for Eqs. (4) and (21), in the following form:

$$-\nabla(G\theta\rho(a)) + m(a) \left[\frac{K(\rho)}{ax(a)} uu + \left(1 - \rho \frac{\langle d_1 \rangle}{d_1(a)} \right) g \right] \rho(a) = 0. \quad (22)$$

In essence, this equation defines the distribution density $\rho(a)$ as a function of coordinates, i.e., it is integrodifferential because G and θ depend on ρ obtained by integrating $\rho(a)$ over the entire range of particle radii.

Next, we consider a binary system consisting of particles with radii a_1 and a_2 , densities d_1 and d_2 , and volume concentrations ρ_1 and ρ_2 in the mixture; $\rho_1 + \rho_2 = \rho$. Introducing the parameters

$$\alpha = a_2/a_1, \quad \beta = d_2/d_1; \quad a^\circ = a_1, \quad \kappa^\circ = \kappa_1. \quad (23)$$

we obtain from Eqs. (5) and (13)

$$\begin{aligned} \langle d_1 \rangle &= \frac{\rho_1 d_1 + \rho_2 d_2}{\rho}, \quad \gamma_2 = \frac{\rho_1 + \alpha^{-1} \rho_2}{\rho_1 + \beta \rho_2}, \\ \gamma_3 &= \frac{\rho_1 + \alpha^{-5/2} \beta^{-1/2} \rho_2}{(\rho_1 + \alpha^{-3} \rho_2)^{1/2} (\rho_1 + \beta \rho_2)^{1/2}}. \end{aligned} \quad (24)$$

These expressions completely define M and γ in conformity with Eq. (16) and, consequently, the temperature in Eq. (20). The partial pressures of particles of two kinds are as follows in this case: $p_j = G\theta(\rho_j/\sigma_j)$, $j = 1, 2$.

Of interest is the effect of incorporating particles, differing in their properties from the initial particles, into the system on the rate of pseudoturbulent pulsations, with the total volume concentration of the dispersed phase

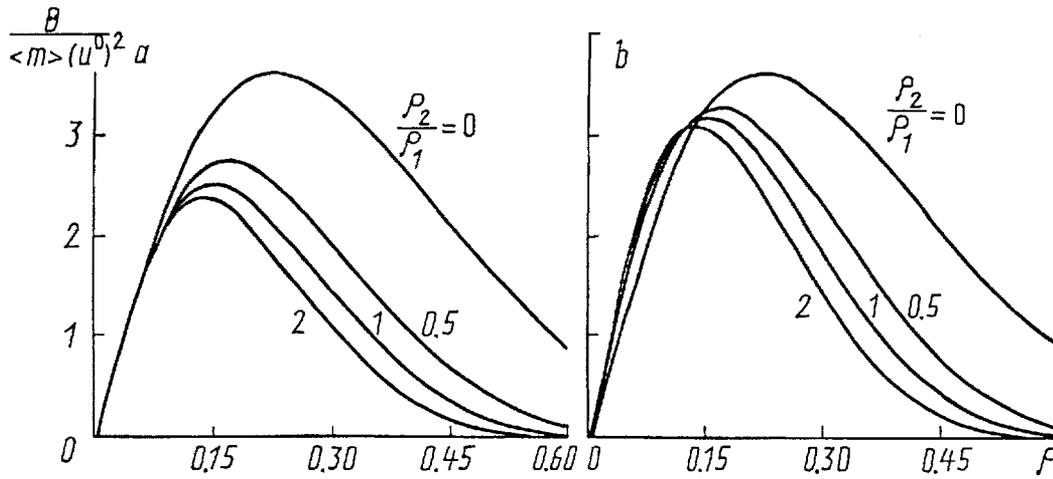


Fig. 1. Dimensionless temperature of pulsations versus concentration on incorporating fine (a; $\alpha = 2^{-1/3}$, $\beta = 1$) and lighter particles (b; $\alpha = 1$, $\beta = 0.75$); the figures at the curves indicate ρ_2/ρ_1 values for $\rho_1 + \rho_2 = \rho$.

being preserved. This information is illustrated in Fig. 1. As a reference state, a homogeneous state of the fluidized bed of the particles of the first kind is used, in which the average gas velocity in the spaces between the particles is

$$u = u_0 = (\epsilon a_1 \kappa_1 g / K)^{1/2}. \quad (25)$$

The temperature in this homogeneous state is described by the curves with the values $\rho_2/\rho_1 = 0$ in Fig. 1; these curves coincide, naturally, with those in Fig. 2 from [21]. The other curves in Fig. 1 pertain to the states in which there are smaller (Fig. 1a) or lighter (Fig. 1b) particles in different concentration whereas the gas velocity and the total concentration of the disperse phase remain the same (but such states are not, of course, macroscopically homogeneous). Obviously, addition of smaller particles virtually does not affect the pulsation rate within the low concentration range but leads to a very pronounced attenuation of pulsations in concentrated systems; the more pronounced, the higher the concentration is. The addition of lighter particles exerts the same effect at high concentrations. But at low concentrations, on the contrary, it favors some intensification of pulsations.

In the case of a binary system, Eq. (22) leads to two coupled equations in which ρ_1 and ρ_2 are unknown variables. Bearing in mind that the fluidized bed is stationary and directing the z axis downward, we may represent these equations ($j = 1, 2$) as

$$\frac{d}{dz} (\rho_j G \theta) + m_j \left[\frac{K}{a_j \kappa_j} u^2 - \left(1 - \frac{\rho_1 d_1 + \rho_2 d_2}{d_j} \right) g \right] \rho_j = 0. \quad (26)$$

The functions $K(\rho)$ and $G(\rho)$ are defined, respectively, in Eqs. (4) and (21), while $\theta(\rho)$ is calculated from Eq. (20) with account for Eqs. (7), (13), (16), and (24).

Particle Distribution along the Height of a Binary Bed. We will consider a stationary fluidized bed containing particles of two kinds; the particles of both kinds are, in the mean, motionless. The state of the bed is fully defined by the prescribed total flow rate $Q = \epsilon u = \text{const}$ of a fluidizing gas per unit area of a gas-distribution grid. We may show (see below) that the bed has a distinct upper boundary at which the quantities ρ_1 and ρ_2 simultaneously vanish. Choosing the reference point at this boundary, we introduce the dimensionless coordinate and the parameter

$$z' = \frac{z}{C a_1 \kappa_1}, \quad q = \frac{Q^2}{a_1 \kappa_1 g}, \quad C = 1,17 \cdot 10^{-3}. \quad (27)$$

In this notation Eq. (26) may be written as

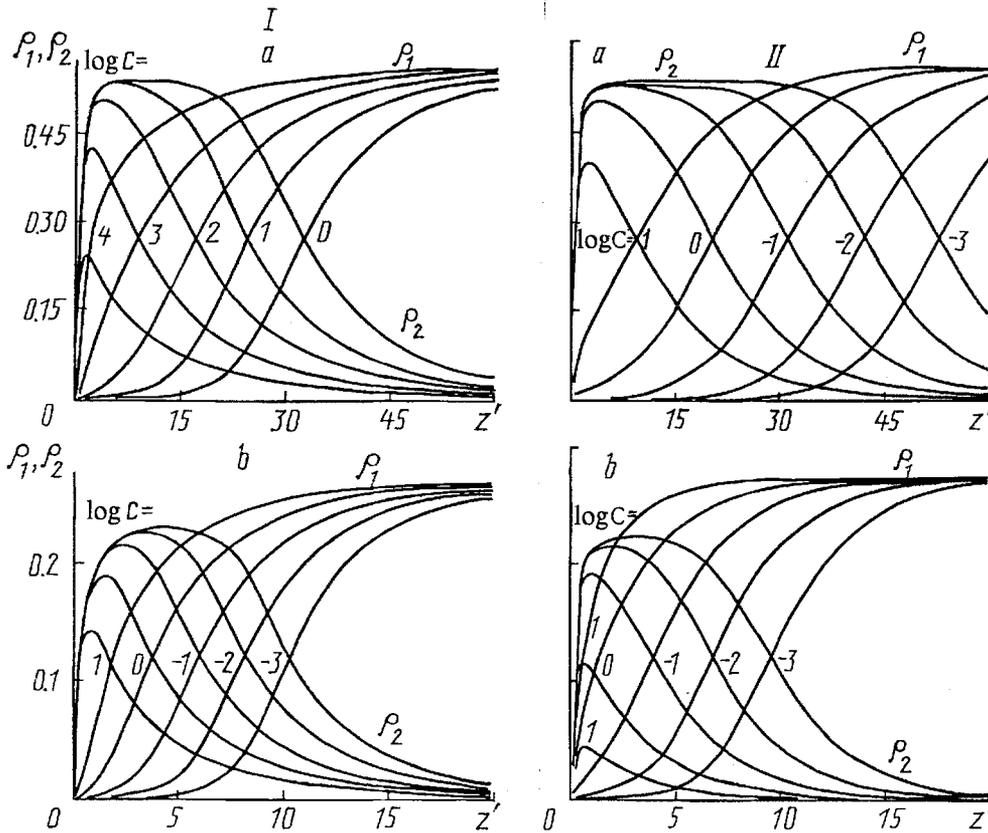


Fig. 2. Particle distribution with respect to the height of a binary bed (I, $\alpha = 2^{-1/3}, \beta = 1$; II, $\alpha = 1, \beta = 0.75$) for $q = 0.1$ (a) and $q = 1$ (b); the figures at the curves indicate $\log c$ values.

$$\begin{aligned} \frac{d}{dz'} \left(\rho_1 \frac{G \langle \rho'^2 \rangle}{\varepsilon^2} P \right) &= \left[\frac{1 - (\rho_1 + \beta \rho_2)}{q} - \frac{K}{\varepsilon^2} \right] \rho_1, \\ \frac{d}{dz'} \left(\rho_2 \frac{G \langle \rho'^2 \rangle}{\varepsilon^2} P \right) &= \alpha^3 \left[\frac{\beta - (\rho_1 + \beta \rho_2)}{q} - \frac{K}{\alpha \varepsilon^2} \right] \rho_2, \\ P &= \left(\frac{\rho_1 + \alpha^{-1} \rho_2}{\rho_1 + \alpha^{-5/2} \beta^{-1/2} \rho_2} \right)^2 \left(\frac{1}{\varepsilon} + \frac{1}{2} \frac{d \ln K}{d \rho} + \frac{\varepsilon^2}{2Kq} \frac{\rho_1 + \beta \rho_2}{\rho_1 + \alpha^{-1} \rho_2} \right)^2. \end{aligned} \quad (28)$$

A homogeneous fluidized state of particles of the j -th kind is possible if for the given Q the equation

$$\varepsilon = \frac{K(\rho)}{a_j \kappa_j g} \left(\frac{Q}{\varepsilon} \right)^2 \quad (29)$$

is solved for ρ in the interval $0 < \rho < \rho_*$, where $\rho_* \approx 0.60$ is the concentration of the close-packing state. For homogeneous states of the fluidized binary mixture to exist, the system of equations

$$\alpha [\beta - (\rho_1 + \beta \rho_2)] = 1 - (\rho_1 + \beta \rho_2) = \frac{K(\rho)}{a_1 \kappa_1 g} \left(\frac{Q}{\varepsilon} \right)^2 \quad (30)$$

must have solutions satisfying the conditions $\rho_1 > 0$, $\rho_2 > 0$, $\rho \leq \rho_*$. As is seen, it has no such solutions for $\alpha < 1$, $\beta < 1$ and for $\alpha > 1$, $\beta > 1$.

Now we consider asymptotic forms of solutions of (28) in the vicinity of the point z' , where $\rho_1 \ll \rho_2 \ll 1$. After simple manipulations we arrive at equations approximately valid in the region considered:

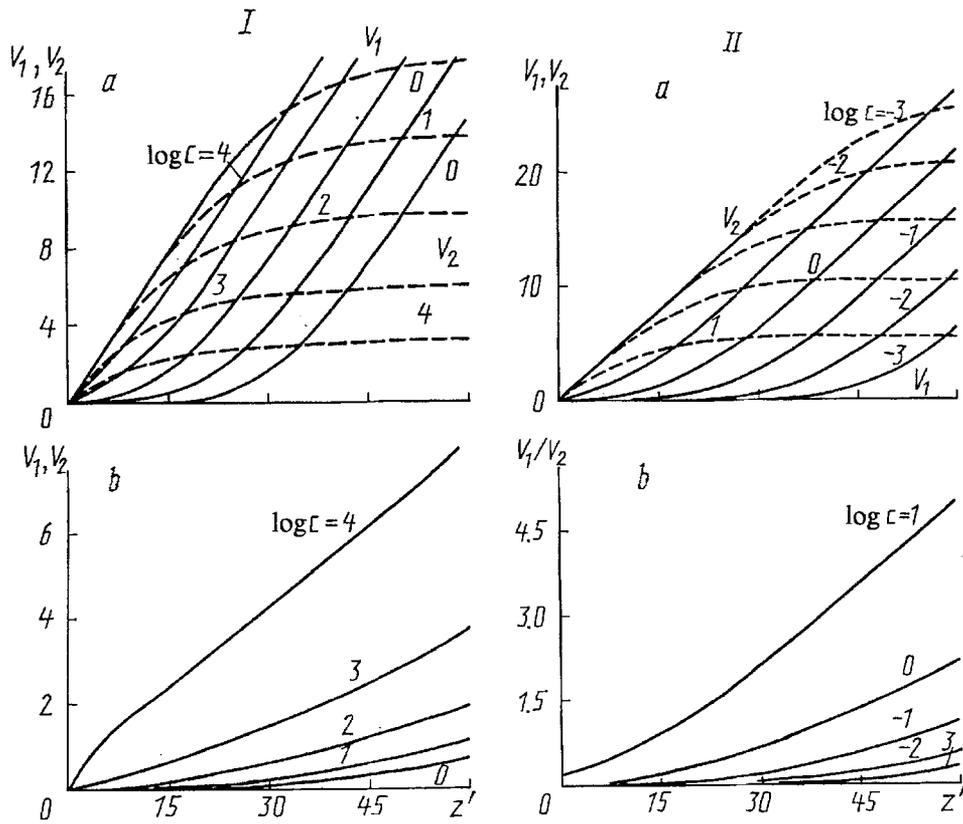


Fig. 3. Contents of fractions (a) and composition (b) of a binary bed with $\alpha = 2^{-1/3}, \beta = 1$ (I) and with $\alpha = 1, \beta = 0.75$ (II) for $q = 0.1$ versus dimensionless height h' ; the figures at the curves indicate $\log c$ values; the solid and dashed curves in Fig. 3a pertain to V_1 and V_2 , respectively.

$$\frac{d}{dz'} (\rho_1 \rho_2^{4/3}) \approx \frac{1,64}{\alpha^3 \beta} \left(\frac{1}{q} - K_0 \right) \rho_1, \quad K_0 = K(0) = \frac{3\xi}{8}, \quad (31)$$

$$\frac{d}{dz'} (\rho_2^{7/3}) \approx 1,64 \left(\frac{1}{q} - \frac{K_0}{\alpha\beta} \right) \rho_2.$$

At small z' , the solution of these equations is

$$\rho_2 \approx \left[0,94 \left(\frac{1}{q} - \frac{K_0}{\alpha\beta} \right) \right]^{3/4} (z')^{3/4}, \quad (32)$$

$$\rho_1 \approx c (z')^s, \quad s = \frac{7}{4\alpha^3\beta} \frac{1/q - K_0}{1/q - K_0/\alpha\beta} - 1,$$

where the constant c can be arbitrary. At small z' , the condition $\rho_1 < \rho_2$ is fulfilled provided

$$\frac{1}{q} - K_0 > \alpha^3\beta \left(\frac{1}{q} - \frac{K_0}{\alpha\beta} \right). \quad (33)$$

This inequality implies that it is easier to fluidize particles of the second kind than those of the first kind. If the inequality is not fulfilled, we change the numbering of the fractions.

The asymptotic forms of Eqs. (32) make it possible to prescribe the initial data for numerical integration Eqs. (28). Distributions $\rho_j(z')$ obtained for different c for particles differing only in size are given for two values of q in Fig. 2 (I). Figure 3 (I) gives the corresponding curves for the integrals

$$V_j = \int_0^{h'} \rho_j(z') dz', \quad j = 1, 2, \quad (34)$$

characterizing the total volume concentration of both fractions in a fluidized bed of dimensionless height h' and these for their ratio describing the fractional composition of the bed for one of the values of q .

If for a prescribed q a mixture with a given composition and a known content of either of the two fractions is fluidized, the parameters c and h' should be determined from the curves in Fig. 3 (I). Furthermore, curves of the type of those in Fig. 2 (I) are employed to determine the distributions of the fractions along the height of the bed. Analogous information for particles differing only in their density is presented in Figs. 2 (II) and 3 (II).

A sufficiently high binary fluidized bed at comparable contents of particles of both fractions breaks down into several zones. In the lower and upper parts of the bed regions are formed in which almost monodisperse systems are, in fact, fluidized, with their particles being fluidized, respectively, more difficultly or easily. These regions, practically not differing from monodisperse fluidized beds, are separated by a mixing zone that has particles of both kinds. The thickness of this zone may attain, as seen in Fig. 2 (I, III), several tens of units of change in the dimensionless coordinate z' . Since for gas-fluidized beds $Ck \sim 1$, then according to Eq. (27) it amounts to several tens of diameters of the particles. In this case, the indicated thickness decreases markedly with increasing gas flow rate, which is fully consistent with the experimental inferences about enhanced separation of the particles as the fluidization number increases. (This inference is by no means trivial since with increase in this number one might expect intensification of mixing processes and, as a consequence, thickening of the mixing zone.)

For reasons of space we cannot offer a more complete analysis of the distributions of different particles along the height of both the binary and the polyfractional fluidized beds, following, correspondingly, from Eqs. (28) and their analogs obtained from Eq. (22). However, it is reasonably safe to suggest that the results of such analysis are in good qualitative agreement with practically all the experimental data in [10-19].

In conclusion, it may be noted that the theory of this work is restricted by the implicit assumption that the structure of the bed is close to a homogeneous one. This means that we have completely ignored mixing due to particle transfer by rising bubbles as well as to circulating flows in the bed, whose effect may be considerable (see, e.g., [13]).

NOTATION

A, B , coefficients in Eq. (12); a , particle radius; b , parameter in (17); C, c , coefficients in Eqs. (19) and (32), respectively; d , density; F , function introduced in (11); f , force acting on a particle; G , function introduced in Eq. (21); g , acceleration due to gravity; h' , dimensionless height of the bed; K , function defined in Eq. (4); k , wave vector; M , function introduced in Eq. (16); m , particle mass; n , numerical concentration of the particles; P , function defined in Eq. (28); p , pressure; Q , gas flow rate; q , parameter introduced in Eq. (27); s , exponent in Eq. (32); u , gas velocity; ζ , resistance coefficient, $\zeta \approx 1/2$; V , integral in (34); w' , pulsation velocity of the particles; x, y , parameters introduced in Eq. (16); z, z' , dimensional and dimensionless vertical coordinates; α, β , ratios of radii and densities of the particles; γ_j and γ , parameters defined in Eqs. (13) and (16), respectively; ϵ , porosity; u^0 , free-fall velocity of the particles; θ , temperature of pulsations (doubled mean energy per degree of freedom); κ , particle-to-gas density ratio; ρ , volume concentration of the particles; σ , particle volume.

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